MECHANICAL PROPERTIES OF SOLID.

ELASTICITY

\# The tendency of an object (Molecule of Solid) to regain its original position, its original Configuration when Deforming force is removed. Up to its Elastic limit.

\# The Tendency to oppose any change in its Shape, Size by a body.

Q Which is More elastic?? Still or Rubber??

Ans. Still, because it come very speedly to its original position.

Q **Why Elasticity?**

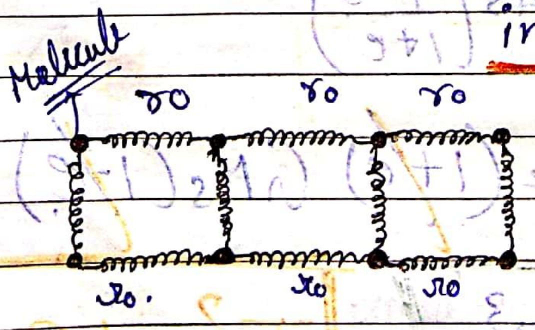
Molecule → Deforming Force (External)

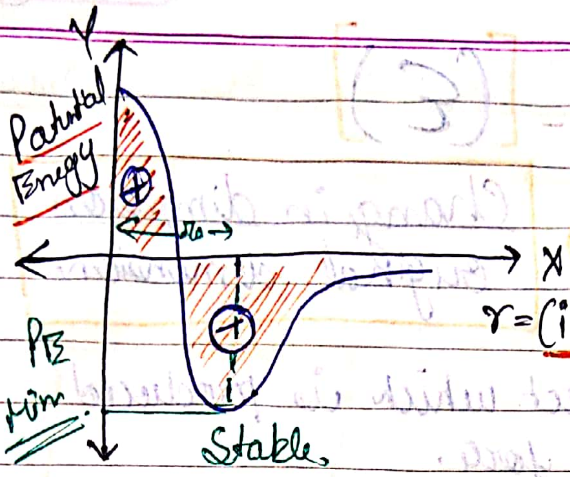
⇓

Restoring Force
intermolecular Force

⇓

Elasticity



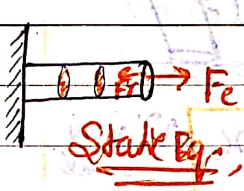


P.E (minimum Stable)

r = (inter molecular distance)

STRESS :-

The internal restoring force per unit cross sectional Area.



Restoring Force = Deforming Force

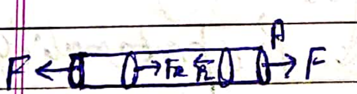
$\boxed{F_R = F_{ext}}$ 99% Case

$$\boxed{\sigma = \frac{F_R}{A_{\text{cross sect.}}}} \quad \frac{N}{m^2}$$

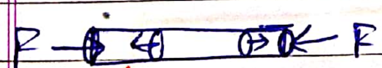
~~Scalar q.~~ (Tensor)

TYPES :-

LONGITUDINAL



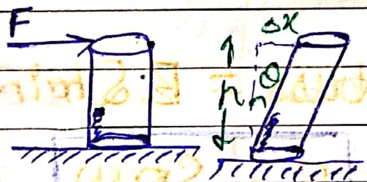
Tensile Stress.



Compressive.

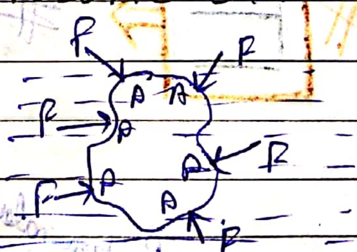
$$\boxed{\sigma = \frac{F}{A_{\text{Cross sect}}}}$$

SHEARING / TANGENTIAL



$$\boxed{\frac{F_t}{A} = \sigma}$$

VOLUME STRESS



$$\boxed{\sigma = \frac{F}{A} = P}$$

$$\boxed{\sigma = \Delta P}$$

$(P - P_0)$ 2 pressure

Ex Plastic Ball

$\sin\theta \to$ small $\to \theta$

$\tan\theta \to$ small $\to \theta$

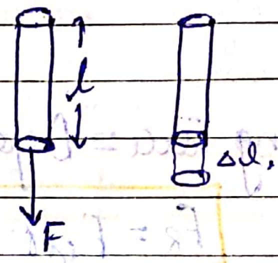
Always +ve STRAIN :- $[\varepsilon]$

$\rightarrow \boxed{\dfrac{\text{Change in dimension}}{\text{Original dimension}}}$ No unit

\# It is an effect which is produced by deforming force.

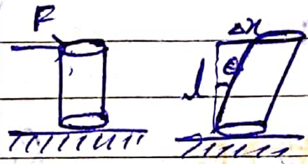
TYPES :-

LONGITUDINAL



$$\boxed{\varepsilon = \frac{\Delta l}{l}}$$

SHEAR STRAIN



$$\boxed{\varepsilon = \frac{\Delta x}{l}}$$ Δx very small

$$\boxed{\frac{\Delta x}{l} = \tan\theta} \to \theta \text{ (radian)}$$

$$\boxed{\varepsilon = \frac{\Delta x}{l} = \theta}$$

VOLUME STRAIN

$$\boxed{-\frac{\Delta V}{V} = \varepsilon}$$

Because it comes to be negative (-ve)

\# $\boxed{E}$ $\Rightarrow$ \# Stress $\propto$ Strain. (limit of Proportionality).

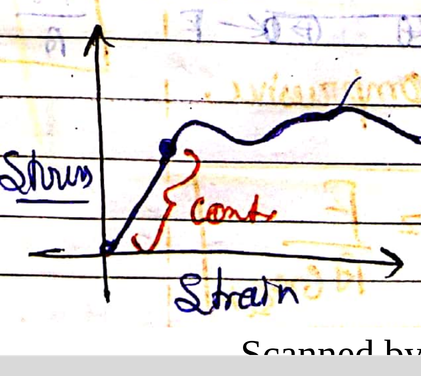
Stress = E Strain

Proportionality constant $\leftarrow$ $\boxed{E = \dfrac{\text{Stress}}{\text{Strain}}}$ $\dfrac{N}{m^2}$

K

$E \Rightarrow$ Modulus of Elasticity

Property of Material.



(i) YOUNG'S MODULUS OF ELASTICITY

$$Y = \frac{\text{longitudinal stress}}{\text{longitudinal strain}}$$

$$\boxed{Y = \frac{Fl}{A\Delta l}}$$

(ii) SHEAR MODULUS OF ELASTICITY

$$G = \frac{\text{Shear Stress}}{\text{Shear Strain}}$$

MODULUS OF RIGIDITY

$$G = \frac{F_t l}{A \Delta x}$$

$$\boxed{G = \frac{F_t}{A\theta}}$$

(iii) BULK MODULUS OF ELASTICITY

$$B = \frac{\text{Volume Stress}}{\text{Volume Strain}}$$

$$B = \frac{F}{A}\left(\frac{V}{-\Delta V}\right)$$

$$\boxed{B = -\frac{\Delta P V}{\Delta V}}$$

COMPRESSIBILITY

$$\boxed{C = \frac{1}{B}}$$

Q. Elongation of a steel bar 1 m long and 1.5 cm² cross sectional Area when subjected to a pull of 1.5×10^4 N. ($Y_s = 2 \times 10^{11}$ N/m²)

a) 0.1 mm b) 0.3 mm c) 0.5 mm d) 0.2 mm

$$Y = \frac{\text{Stress}}{\text{Strain}} = \boxed{\frac{F l}{A \Delta l}}$$

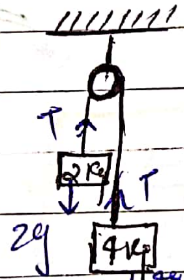
$$\Delta l = \frac{F l}{A Y}$$

$$= \frac{1.5 \times 10^4 \times 1}{1.5 \times 10^{-4} \times 2 \times 10^{11}} = \frac{1}{2} \times 10^{8-11}$$

$$= 0.5 \times 10^{-3} \text{ m}$$

$$\boxed{= 0.5 \text{ mm}}$$

Q.



Wire → Breaking stress $= 2 \times 10^5$ N/m². Find the min Cross sectional Area of wire so that it doesn't break.

a) 2.66 cm² b) 1.66 cm² c) 3.33 cm² d) 1.33 cm²

$$\text{Stress} = \frac{F}{A}$$

$$\boxed{T = \frac{2 m_1 m_2 g}{m_1 + m_2}}$$ Trick to find Tension

$4g - \cancel{T} = 4a$
$\cancel{T} - 2g = 2a$

$2g = 6a$

$$\boxed{a = g/3}$$

$T = 2a + 2g$

$$\boxed{T = \frac{8g}{3}}$$

$$\text{Stress} = \frac{F}{A} = \frac{T}{A}$$

$$2 \times 10^5 = \frac{8g}{3A}$$

$$A = \frac{8g}{3 \times 2 \times 10^5}$$

$$A = \frac{\cancel{8}^{\,4}\times10^{8}}{3\times\cancel{2}\times10^{8}\times10^{4}}$$

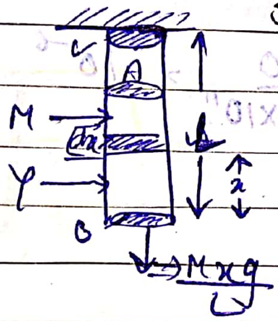
$$A = \frac{4}{3}\times10^{-4}$$

$$A = 1.33\times10^{-4}\,m^2$$

$$\boxed{A = 1.33\,cm^2}$$

Q) Find the extension in a wire of length 'L' Cross Section Area (A), ~~density~~ Mass M & Young's Modulus of Elasticity Y under its own weight??

a) $\frac{MgL}{2AY}$ ✓ b) $\frac{MgL}{3AY}$ c) $\frac{2MgL}{3AY}$ d) $\frac{MgL}{AY}$



$$Y = \frac{Fl}{A\Delta l}$$

$$\Delta l = \frac{Fl}{AY}$$

$L \rightarrow M$

$x \rightarrow \frac{Mx}{L}$

Elongation in Small element 'dx'

$$dl = \frac{F\,dx}{AY}$$

$$\int dl = \int_0^L \frac{Mxg\,dx}{LAY}$$

$$= \frac{Mg}{ALY}\int_0^L x\,dx$$

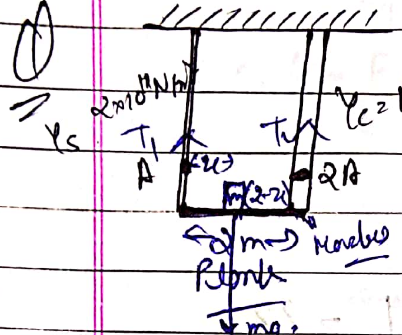
$$\Delta l = \frac{Mg}{ALY}\,\frac{L^2}{2}$$

$$\boxed{\Delta l = \frac{MgL}{2AY}}$$

$$\left(\frac{T_M}{T}\right)^2 = 1 + \frac{Mg}{AY}$$

$$\left(\frac{T_M}{T}\right)^2 - 1 = \frac{Mg}{AY}$$

$$\boxed{\frac{1}{Y} = \left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{Mg}}$$

Q. 

Where should a mass m be kept so that in steel & copper wire, we obtain
(i) equal stress
(ii) equal strain

(ii) a) 1.00 m
b) 0.5 m
c) 0.33 m
(i) d) 1.33 m
e) 1.66 m
f) 0.8 m

(i) $(\text{Stress})_s = (\text{Stress})_c$

$$\frac{T_1}{A} = \frac{T_2}{2A}$$

$$\boxed{T_2 = 2T_1}$$

equilibrium $\tau_{net} = 0$ ($r \times f$)

$$T_1 \times x - T_2(2-x) = 0$$

$$T_1 x - 2T_1(2-x) = 0$$

$$x - 4 + 2x = 0$$

$$3x = 4$$

$$\boxed{x = \frac{4}{3}} \quad \boxed{x = 1.33}$$

$(\text{Strain})_s = (\text{Strain})_c$

$Y = \dfrac{\text{Stress}}{\text{Strain}}$

$\left(\dfrac{\text{Stress}}{Y}\right)_s = \left(\dfrac{\text{Stress}}{Y}\right)_c$

$\left(\dfrac{T}{\cancel{A} Y_s}\right) = \left(\dfrac{T_2}{2A Y_c}\right)$

$\dfrac{T}{\cancel{2\times10^{11}}} = \dfrac{T_2}{\cancel{2\times10^{11}}}$

$\boxed{T_1 = T_2}$.

eq. $T_{net} = 0$.

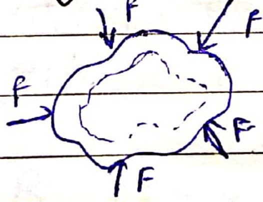
$T_1 x - T_2 (2-x) = 0$

$x - 2 + x = 0$

$2x = 2$

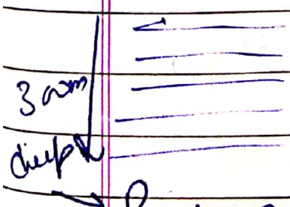
$\boxed{x = 1}$.

Q2 Avg. dept of Indian Ocean is about 3000 m. Calculate the fractional Compression $\frac{\Delta V}{V}$ of water at the bottom of ocean ($g = 10\, m/s^2$)
$\rho_w = 10^3\, kg/m^3$, Bulk Mod of water $= 2.2\times10^9\, N/m^2$



Volume Stress $= \dfrac{F}{A} = P$.

Volume Strain $= -\dfrac{\Delta V}{V}$



$B = \dfrac{\text{Vol. Stress}}{\text{Vol. Strain}} = \dfrac{-PV}{\Delta V}$

$P = h\rho g$.

$= 3000\times10^3\times10 = 3\times10^6\, N/m^2$

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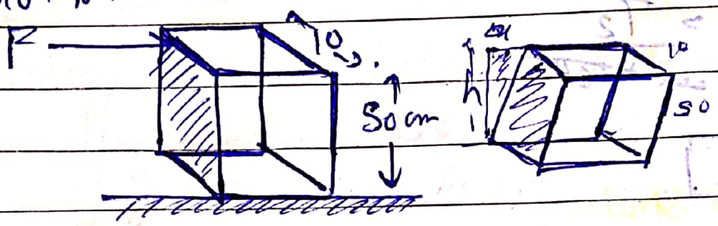
$$\frac{\Delta V}{V} = \frac{P}{B} = \frac{3\times10^{6}}{2.2\times10^{9}}$$

Q. A Square lead slab of side 50 cm & Thickness 10 cm is subjected to a shearing force 9×10^{4} N. How much upper edge will be displaced. If shear modulus of elasticity $G = 5.6\times10^{9}$ N/m² for lead.



$$\text{Shear Stress} = \frac{F}{A} = \frac{9\times10^{4}}{50\times10\times10^{-4}}\ \frac{N}{m^2}$$

$$\text{Shear Strain} \Rightarrow \frac{\Delta x}{50}$$

$$G = \frac{9\times10^{4}}{50\times10\times10^{-4}}$$

$$G = \frac{\quad}{\frac{\Delta x}{50}}$$

$$G\ \Delta x =$$

$$\frac{9\times10^{4}}{50\times10\times10^{-4}}$$

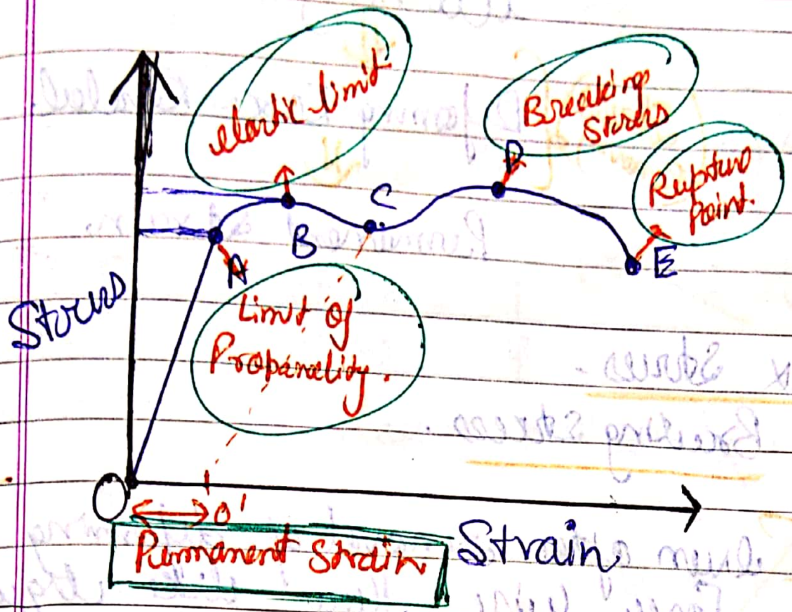
$$\frac{9\times10^{8}}{5\times10} \quad \frac{9\times10^{8}}{8}$$

$50\times1.8\times10^{6}$

9×10^{7}

1.0 × 5 = 9.0

STRESS-STRAIN CURVE.



Stress $\propto$ Strain

$\frac{F}{A}$ $\frac{\Delta l}{l}$

OA → Stress $\propto$ Strain.

A → limit of proportionality.
(Stress)

Slope $= \frac{\Delta y}{\Delta x} = \frac{\text{Stress}}{\text{Strain}}$ = Modulus of Elasticity.

B → elastic limit
(Stress)

(Force is removed uptill B, wire returns to its original state.)

A → B stress $\cancel{\propto}$ strain.

B→D Plastic Region

After B →. Small stress → large strain ↑↑.

Elasticity ✗

(Stress Removed) { Deforming Force Removed
Permanent Strain.

D → Max Stress.
Breaking stress.

✱ { Even after removal of deforming Force wire flows like liquid (thinning of wire ↑↑)

E → Rupture Point

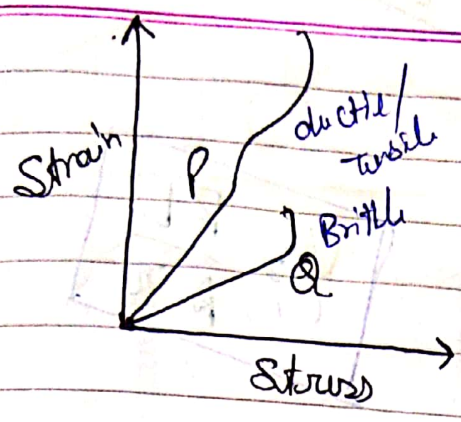
OB → Elastic Region
BD → Plastic Region.

1) DUCTILE / TENSILE.
Plastic Region Large.

2) BRITTLE →
Plastic Region small.

3) ELASTOMER
No Plastic Region.
only have
Elastic Region (Rubber)

Q (18)



a) P has more tensile strength

b) P is More ductile

c) P is More brittle

~~d)~~ Y of P is More than that of Q.

$$\text{Slope} = \frac{\text{Strain}}{\text{Stress}} = \frac{1}{Y}$$

$$(\text{Slope})_P > (\text{Slope})_Q$$

$$\left(\frac{1}{Y}\right)_P > \left(\frac{1}{Y}\right)_Q$$

$$\boxed{Y_Q > Y_P}$$

ENERGY STORED IN A STRECHED WIRE

$$U = \frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{Volume}$$

$$\frac{PE}{Vol.} = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

$$\boxed{E = \frac{\text{Stress}}{\text{Strain}}}$$

$$\boxed{\frac{PE}{Vol.} = \frac{1}{2} \times E \times (\text{Strain})^2}$$

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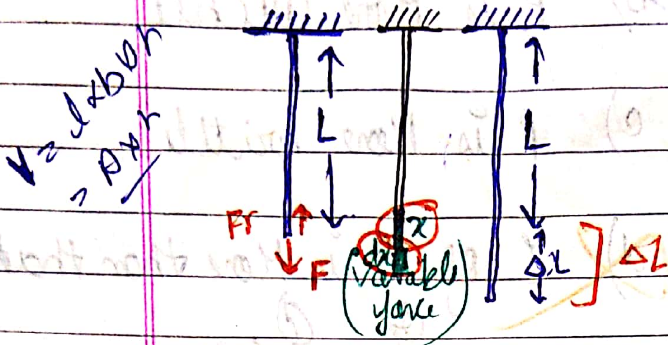
$$\boxed{F = YA\frac{\Delta L}{L}}$$

DERIVATION :-



$$\boxed{Y = \frac{FL}{Ax}}$$

Small Work done in stretching wire by 'dx' amount when it is stretched upto 'x' amount

$$dw = F\,dx.$$

$$\int dw = \int_0^{\Delta L}\left(\frac{YAx}{L}\right)dx.$$

$$W = \frac{YA}{L}\left[\frac{x^2}{2}\right]_0^{\Delta L}$$

$$W = \frac{YA}{L}\frac{(\Delta L)^2}{2}$$

$$W = \frac{YA}{L} \times \frac{(\Delta L)^2}{2} \times \frac{L}{L}$$

$$W = \frac{YA}{L} \times \left(\frac{\Delta L}{L}\right)^2 \times \frac{1}{2}$$

$$\left(\frac{1}{2}\sigma\epsilon\right)$$

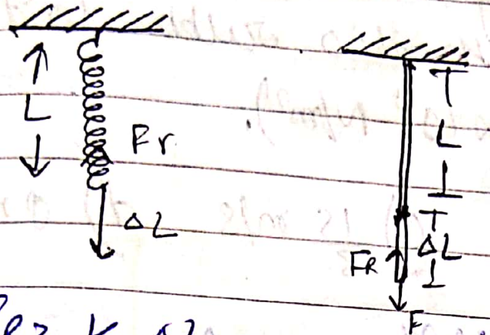
$$W = Y \times \text{volume} \times (\text{Strain})^2 \times \frac{1}{2}$$

$$W = \frac{1}{2} \times (\text{Strain})\,(\text{Strain}\,Y)\ \text{volume}$$

PE
U

$$\boxed{W = \frac{1}{2} \times \text{Stress} \times \text{strain} \times \text{volume}}$$

TRICK :-



$$Y = \frac{FL}{A\,\Delta L}$$

$F_R = K\,\Delta L$

$$F = \left(\frac{YA}{L}\right)\Delta L$$

$$K\cancel{\Delta L} = \left(\frac{YA}{L}\right)\cancel{\Delta L}$$

$$\boxed{K = \frac{YA}{L}}$$ **Most**

Energy stored in a streched spring $= \frac{1}{2}K(\Delta x)^2$

$$= \frac{1}{2} \times \frac{YA}{L}(\Delta L)^2 \times \frac{L}{L}$$

$$= \frac{1}{2} \times Y(AL)\left(\frac{\Delta L}{L}\right)^2$$

$$= \frac{1}{2} \times Y \times \text{vol} \times (\text{Strain})^2$$

$$\boxed{W = \frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{volume}}$$

Q A rubber cord of length 10 cm is stretched upto 12 cm. If cross sectional area of cord is 1 mm², find the velocity of a missile (mass 5g) which is hit upon with this rubber cord ($Y_{Rubb} = 5\times10^{8}$ N/m²).

a) 10 m/s b) 20 m/s c) $\frac{15}{3}$ m/s d) 0 m/s

Tension / Force (T) $= \frac{YA\ell}{L} = 100$ N.

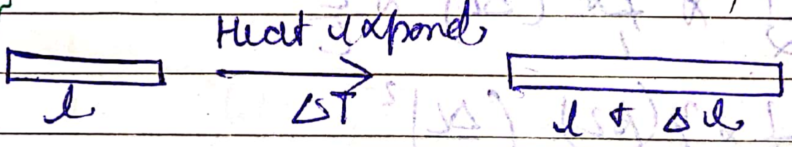
Velocity $V = \sqrt{\frac{Fl}{m}}$

$= \sqrt{\frac{100 \times 2 \times 10^{(-2)}}{5 \times 10^{(-3)}}}$

$= 20$ m/s

THERMAL STRESSES & STRAIN

Rod free to deform



$\boxed{\Delta l = l\alpha\Delta T}$

(alpha) [Thermal coefficient of linear expansion]

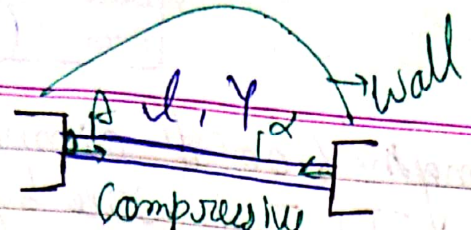
$\alpha \Rightarrow$ unit temp increase

$\Downarrow$

unit length me

$\Downarrow$

kitna change hoga



Temp rise → ΔT

New length $= l + \Delta l$

$= l + l\alpha\Delta T$

Not free to deform

$$\boxed{\text{Thermal Strain} = \frac{\Delta l}{l}}$$

[original leng → 'l']

Thermal Stress $= Y \text{ strain} = \frac{Y\Delta l}{l}$

$\frac{\text{Stress}}{\text{Strain}} = Y$

$\text{Stress} = \frac{Y(l\alpha\Delta T)}{l}$

$$\boxed{\text{Thermal Stress} = Y\alpha\Delta T}$$

How much Force :-

$\text{Stress} = \frac{F}{A}$

$F = \text{Stress} \times \text{Area}$

$$\boxed{F = Y\alpha A\Delta T}$$

Q A Steel rod of length 6.0 m & diameter 20mm is fixed as shown. If Temp. rise by 80°C find the stress in rod. $Y = 2\times10^{6}$ kg/cm² $\alpha = 12\times10^{-6}$ per °C person

$\Delta T = 80°C$

$\Delta l = l\alpha\Delta T$

$\text{Strain} = \frac{\Delta l}{l} = \frac{l\alpha\Delta T}{l} = \alpha\Delta T$

$\text{Stress} = Y\,\text{Strain}$

$= Y\alpha\Delta T$

$= 2\times10^{6}\frac{kg}{cm^2}\times12\times10^{-6}\times80$

$= 1920\, kg/cm^2$

Q2 In the previous Q. Agar that :— if it is allowed to yield (expand by 1 mm)

a) 1920
b) More
c) Less.

Kitna expand Karna Chahiye tha.

$\Delta l = l\alpha\Delta T$

$\boxed{\text{Strain} = \frac{\Delta l - 1mm}{l}}$

$\text{Stress} = Y\,\text{Strain}$

$= 2\times10^{6}\left(\frac{l\alpha\Delta T - 1mm}{l}\right)$

$= 2\times10^{6}\left(\alpha\Delta T - \frac{0.1cm}{600cm}\right)$

$= 2\times10^{6}\left(12\times10^{-6}\times80 - \frac{0.1}{600}\right).$

$$\boxed{\text{Stress} = \gamma\, \alpha\, \Delta T}$$

$= 1920 - 2\times10^6 \times \frac{1}{6000}$

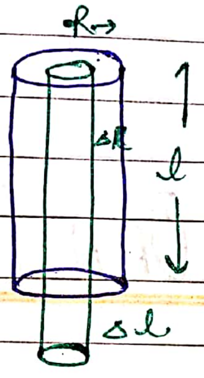
$= 1920 - \frac{2\times10^3}{6} = 1920 - \frac{1000}{3}$

$= 1920 - 333.3$

$= 1586.67 \text{ kg/cm}^2$

POISSON'S RATIO.

$$\boxed{\sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}}}$$



$$\boxed{\sigma = \ominus \frac{\Delta R/R}{\Delta l/l}}$$

+ve

Q)  → It's length increase by 1%.
if $\sigma = \frac{1}{4}$, find the % change in its vol.

a) 1% increase

b) 1% decrease

c) 0.5% increase

d) 0.5% Decrease

$$V = \pi R^2 L.$$

inc.

$$\left(\frac{\Delta V}{V}\times 100\right) = 2\left(\frac{\Delta R}{R}\times 100\right) + \left(\frac{\Delta L}{L}\times 100\right)$$

$$= -2\times\frac{1}{4}\left(\frac{\Delta l}{l}\times 100\right) + \left(\frac{\Delta L}{L}\times 100\right)$$

$$\sigma = -\frac{\Delta R/R}{\Delta l/l}$$

$$\boxed{\frac{\Delta R}{R} = -\left(\frac{\Delta l}{l}\cdot\frac{1}{4}\right)}$$

$$= -2\times\frac{1}{4}(1\%) + 1\%$$

$$= 1 - \frac{1}{2} = 0.5\%$$

CHANGE IN DENSITY OF A LIQUID

⇓

Due to Pressure.

$$\boxed{\rho\uparrow \Rightarrow P\uparrow}$$

Practical (Non-Ideal liquid) # ideal fluid → Incompressible ↓ [Density is same]

$$\rho = \frac{m}{V}$$

$$\rho \propto \frac{1}{V}$$

$$\rho \rightarrow \rho'$$

$$V \rightarrow V + dV \ (-)$$

$$\frac{\rho'}{\rho} = \frac{V}{V+dV}$$

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Bulk Mod.

$$B = -\frac{dP}{\left(\frac{dv}{v}\right)}$$

$$= \frac{1}{1 + \frac{dv}{v}}$$

$$\boxed{\rho' = \rho\left(\frac{1}{1 - \frac{dP}{B}}\right)}$$

$P\uparrow \Rightarrow dP = +ve.$

$\left(1 - \frac{dP}{B}\right)\downarrow \Rightarrow \rho\uparrow.$ //

$\boxed{\text{FINISH}}$